STABILITY OF RECTANGULAR FGM PLATES USING NEW FOUR VARIABLE REFINED PLATE THEORY UNDER VARIOUS TYPES OF THERMAL LOADING

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ABSTRACT

Here, thermoelastic stability of functionally graded rectangular plates is investigated using the new four variable refined plate theory. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories. The theory presented is variationally consistent, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying shear stress free surface conditions. Temperature field is assumed to be a uniform distribution over the plate surface and varied in thickness direction only. Material properties are graded in the thickness direction according to simple power law distribution. For the numerical illustrations, aluminum/alumina is considered as functionally graded material. The variation in critical buckling load is highlighted considering gradient index, temperature, the relative thickness.

Keywords: FGM plates; Thermal buckling; New four-variable refined plate theory.

TRODUCTION

onally graded (FG) plates have received considera- ;earch efforts, recently. These structures are inhomo- ɹs composites fabricated from a mixture of metal and ic with smooth and continuous variation of material ties through the thickness. In comparison to isotropic ninated plates they have reduced thermal stresses and concentrations and have the capability of withstand- gh temperature gradient environments without losing ıral integrity. These advantages together with high th and light weight have made functionally graded als a suitable replacement for conventional materi- aeronautical vehicles as thermal barriers. Therefore, ng the vibration characteristics of FG structures un- ge amplitude thermo-mechanical loads is of para- importance.

researches are conducted on the subject of thermal echanical instability of plate- like structures. Devel- t of functionally graded materials, as a branch of new als, has necessitated more investigations on this sub- mong the primary researches in this field, one may the works of Bouazza et al for both mechanical and ıl buckling cases [1–4]. Javaheri and Eslami [5,6] pre- the thermal buckling analysis of functionally graded supported rectangular plates based on classical and rder shear deformation plate theories. They obtained form solutions for four types of thermal loads. Feld- ıd Aboudi [7] carried out elastic buckling analysis of nally graded plates subjected to axial load and also gated the optimal spatial distribution of the volume n to improve buckling resistance.

sponse of a functionally graded ceramic–metal plate vestigated by Praveen and Reddy using a finite ele- nodel that accounts for the transverse shear strains, inertia, and moderately large rotations in the Von n sense [8]. Sofiyev [9] studied the buckling anal- f circular truncated conical and cylindrical shells ıcted from FGM subjected to combined axial exten- ads and hydrostatic pressure. He analytically found tical combined buckling loads for above structures r without elastic foundations. Zenkour and Sobhy ;ed the sinusoidal shear deformation plate theory to e critical buckling temperature difference of various etric FGM sandwich plates subjected to three types mal loadings, i.e. uniform, linear and non-linear dis- ən through-the-thickness.

ly, Chen et al [11] presented the thermal buckling s is performed on hybrid functionally graded plates with an arbitrary initial stress, The results indicate existence of an initial stress may significantly affect ıaviors of FGPs. Akbarzadeh et al. [12] performed an s of coupled thermo-elasticity of simply supported tes based on the Reddy's TSDT. The plate was subjected to lateral thermal shock of step function type on the lower side and upper side of the plate having convection with the ambient. The material properties of the FG plate, except Poisson's ratio, were assumed to be graded in the thickness direction according to a power-law distribution in terms of the volume fractions of the constituents. Ghannadpour et al. [13] studied the buckling analysis of functionally graded plates under thermal loadings using the finite strip method, the solution is obtained by the minimization of the total potential energy and solving the corresponding eigenvalue problem. Here, a two variable refined plate theory (RPT) was first developed for isotropic and orthotropic plates by Shimpi, Shimpi and Patel [14, 15] is extended to analyze the thermoelastic stability behavior of functionally graded rectangular plates. The present theory satisfies equilibrium conditions at the top and bottom faces of the plate without using shear correction factors. Navier solution is used to obtain the closed-form solutions for simply supported FG plates. A detailed investigation is carried out to bring out the influences of power law index of functionally graded materials and geometric parameters.

2. PROBLEM FORMULATION

2.1 Material properties

Consider a rectangular plate of total thickness h as shown in Fig. 1. The plate FGM is made of of ceramic and metal, the material properties of FGM vary continuously across the thickness according to the following equations, which are the same as the equations proposed by Praveen and Reddy [8]:

$$E(z) = E_m + E_{cm} V_f(z) \qquad E_{cm} = E_c - E_m \qquad (1)$$
$$\nu(z) = \nu_0$$

For simplicity, Poisson's ratio of plate is assumed to be constant in this study for that the effect of Poisson's ratio on deformation is much less than that of Young's modulus [16]. where subscripts m and c refer to properties of metal and ceramics, respectively, and Vf (z) is

volume fraction of the constituents which can mostly be defined by power–law functions [2- 8]. For power-law FGM, volume fraction function is expressed as

$$V_f(z) = (z/h + 1/2)^k \qquad (2)$$

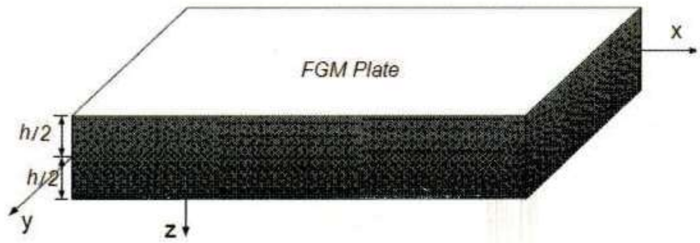

Fig. 1. Typical FGM rectangular plate.

2.2. Higher-order plate theories with five unknown functions

The displacements of a material point located at (x, y, z) in the plate may be written as

$$U(x,y,z)=u_0(x,y)-z\frac{\partial w_0}{\partial x}+\Psi(z)\theta_x$$
$$V(x,y,z)=v_0(x,y)-z\frac{\partial w_0}{\partial y}+\Psi(z)\theta_y \quad (3)$$
$$W(x,y,z)=w_0(x,y)$$

where U,V and W are the displacements in the x, y, and z directions, u0, v0 and w0 are the midplane displacements, and θ x and θ y are the rotations of the yz and xz planes due to bending, respectively. Ψ(z) represents shape function determining the distribution of the transverse shear strains and stresses along the thickness. The displacement field of the classical thin plate theory (CPT) is obtained easily by setting Ψ(z) = 0. The displacement of the first-order shear deformation plate theory (FSDPT) is obtained by setting Ψ(z) = z [17–19]. Also, the displacement of parabolic shear deformation plate theory (PSDPT) of Reddy [20] is obtained by setting

$$\Psi(z)=z\left(1-\frac{4z^2}{3h^2}\right) \quad (4)$$

The sinusoidal shear deformation plate theory (SSDPT) of Touratier [21] is obtained by setting

$$\Psi(z)=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \quad (5)$$

In addition, the exponential shear deformation plate theory (ESDPT) of Karama et al. [22] is obtained by setting

$$\Psi(z)=ze^{-2(z/h)^2} \quad (6)$$

2.3. Present new hyperbolic shear deformation theory

Unlike the other theories, the number of unknown functions involved in the present refined hyperbolic shear deformation theory is only four, as against five incase of other shear deformation theories [23–26]. The theory presented is variation- ally consistent, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying shear stress free surface conditions.

2.3.1. Assumptions of the present plate theory

Assumptions of the present theory are as follows:

(i) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

(ii) The transverse displacement W includes two components of bending wb and shear ws

Both these components are functions of coordinates x and y.

$$W(x,y,z)=w_b(x,y)+w_s(x,y) \quad (7)$$

(iii) The transverse normal stress σ z is negligible in comparison with in-plane normal stresses σ_x and σ_y.

(iv) The in-plane displacements U and V consist of extension, bending, and shear components.

$$U=u+u_b+u_s \text{ and } V=v+v_b+v_s \quad (8)$$

The bending components uband vbare assumed to be similar, respectively, to the displacements given by the classical plate theory. Therefore, the expression for ub and vb can be given as

$$u_b=-z\frac{\partial w_b}{\partial x} \text{ and } v_b=-z\frac{\partial w_b}{\partial y} \quad (9)$$

– The shear components us and vs give rise, in conjunction with ws , to the parabolic variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses σ_{xz}, σ_{yz} through the thickness of the plate in such a way that shear stresses σ_{xz}, σ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as

$$u_s=-f(z)\frac{\partial w_s}{\partial x}, \ v_s=-f(z)\frac{\partial w_s}{\partial y} \quad (10)$$

2.3.2. Kinematics and constitutive equations

Based on the assumptions made in the preceding section, the displacement field of present theory can be obtained using Eqs. (7)–(10) as

$$U(x,y,z)=u_0(x,y)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \quad (11)$$
$$V(x,y,z)=v_0(x,y)-z\frac{\partial w_b}{\partial y}-f(z)\frac{\partial w_s}{\partial y}$$
$$W(x,y,z)=w_b(x,y)+w_s(x,y)$$

Where

$$f(z)=z-\Psi(z) \text{ and } \Psi(z)=z\left(1-\frac{4z^2}{3h^2}\right) \quad (12)$$

It should be noted that unlike the first-order shear deformation theory, this theory does not require shear correction factors. The kinematic relations can be obtained as follows

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 + zk_x^b + f(z)k_x^s \\
\varepsilon_y &= \varepsilon_y^0 + zk_y^b + f(z)k_y^s \\
\gamma_{xy} &= \gamma_{xy}^0 + zk_{xy}^b + f(z)k_{xy}^s \\
\gamma_{yz} &= g(z)\gamma_{yz}^s \\
\gamma_{xz} &= g(z)\gamma_{xz}^s \\
\varepsilon_z &= 0
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
&\varepsilon_x^0 = \frac{\partial u}{\partial x},\ k_x^b = -\frac{\partial^2 w_b}{\partial x^2},\ k_x^s = -\frac{\partial^2 w_s}{\partial x^2} \\
&\varepsilon_y^0 = \frac{\partial v}{\partial y}, k_y^b = -\frac{\partial^2 w_b}{\partial y^2}, k_y^s = -\frac{\partial^2 w_s}{\partial y^2} \\
&\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},\ k_{xy}^b = -2\frac{\partial^2 w_b}{\partial x \partial y}, k_{xy}^s = -2\frac{\partial^2 w_s}{\partial x \partial y} \\
&\gamma_{yz}^s = \frac{\partial w_s}{\partial y}, \gamma_{xz}^s = \frac{\partial w_s}{\partial x} \\
&g(z) = 1 - f'(z), f'(z) = \frac{df(z)}{dz}
\end{aligned} \tag{14}$$

For elastic and isotropic FGMs, the constitutive relations can be written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha T \\ \varepsilon_y - \alpha T \\ \gamma_{xy} \end{Bmatrix} \tag{15}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{16}$$

Where $(\sigma_x,\sigma_y,\tau_{xy},\tau_{yz},\tau_{zx})$ and $(\varepsilon_x,\varepsilon_y,\gamma_{xy},\gamma_{yz},\gamma_{zx})$ are the stress and strain components,

respectively. Using the material properties defined in Eq. (1), stiffness coefficients,Qij can be expressed as

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, Q_{12} = \nu Q_{11}\ ,\ \ Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)} \tag{17}$$

2.3. Governing equations

The strain energy of the plate can be written as

$$U = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij} dV = \frac{1}{2}\int_V \left(\sigma_x(\varepsilon_x - \alpha T) + \sigma_y(\varepsilon_y - \alpha T) + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{xz}\gamma_{xz}\right) dV \tag{18}$$

The principle of virtual work for the present problem may be expressed as follows:

$$\iint \Big[N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + \bar{M}_x^b \delta k_x^b + \bar{M}_y^b \delta k_y^b + \bar{M}_{xy}^b k_{xy}^b + M_x^s k_x^s + M_y^s k_y^s + M_{xy}^s k_{xy}^s + Q_{yz}\gamma_{yz} + Q_{xz}\gamma_{xz}\Big] dx dy = 0 \tag{19}$$

where (N_x,N_y,N_{xy}) denote the total in-plane force resultants, (M_x^b,Mb_y^b,Mb_{xy}^b), (M_x^s,Mb_y^s,Mb_{xy}^s) denote the total moment resultants and (Q_{xz},Q_{yz}) are transverse shear stress resultants and they are defined as

$$\begin{Bmatrix} N_x, N_y, N_{xy} \\ M_x^b, M_y^b, M_{xy}^b \\ M_x^s, M_y^s, M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \tag{20}$$

$$(\Omega_{xz}, \Omega_{yz}) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g(z) dz \tag{21}$$

Substituting Eq. (15) and into Eq. (20), Eq. (16) and into Eq. (21) and integrating through the thickness of the plate, the stress resultants are given as

$$\begin{Bmatrix} \{N\} \\ \{M^b\} \\ \{M^s\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [B^s] \\ [B] & [D] & [D^s] \\ [B^s] & [D^s] & [H^s] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{k^b\} \\ \{k^s\} \end{Bmatrix} - \begin{Bmatrix} \{N^T\} \\ \{M^{bT}\} \\ \{M^{sT}\} \end{Bmatrix} \tag{22}$$

$$\begin{Bmatrix} Q_{yz} \\ Q_{xz} \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \tag{23}$$

where

$$\begin{aligned}
&\{N\} = \{N_x, N_y, N_{xy}\}^t\ ,\ \{M^b\} = \{M_x^b, M_y^b, M_{xy}^b\}^t\ , \{M^s\} = \{M_x^s, M_y^s, M_{xy}^s\}^t \\
&\{N^T\} = \{N_x^T, N_y^T, 0\}^t\ ,\ \{M^{bT}\} = \{M_x^{bT}, M_y^{bT}, 0\}^t,\ \{M^{sT}\} = \{M_x^{sT}, M_y^{sT}, 0\}^t \\
&\{\varepsilon^0\} = \{\varepsilon_x^0; \varepsilon_y^0; \gamma_{xy}^0\}^t;,\ \{k^b\} = \{k_x^b, k_y^b, k_{xy}^b\}^t,\ \{k^s\} = \{k_x^s, k_y^s, k_{xy}^s\}^t
\end{aligned} \tag{24}$$

and A_{ij} , B_{ij} , etc. are the plate stiffness, defined by

$$\begin{aligned}
&(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2, f(z), zf(z), (f(z))^2) dz \quad (i, j = 1,2,6) \\
&A_{ij}^s = \int_{-h/2}^{h/2} Q_{ij}(g(z))^2 dz \qquad (i, j = 4,5)
\end{aligned} \tag{25}$$

The stress and moment resultants, $\{N^T\}$, $\{M^{bT}\}$, $\{M^{sT}\}$due to thermal loading are defined by

$$\{N^T\} = \int_{-h/2}^{h/2} \{\beta\} T\, dz\ ,\ \ \{M^{bT}\} = \int_{-h/2}^{h/2} \{\beta\} Tz\, dz,\ \ \{M^{sT}\} = \int_{-h/2}^{h/2} \{\beta\} T f(z)\, dz \tag{26}$$

where

$$\{\beta\} = \begin{Bmatrix} (Q_{11} + Q_{12})\alpha \\ (Q_{12} + Q_{22})\alpha \\ 0 \end{Bmatrix} \tag{27}$$

The stability equations of the plate may be derived by the adjacent equilibrium criterion. Assume that the equilibrium state of the FGM plate under thermal loads is defined in terms of the displacement components $(u_0^0, v_0^0, w_b^0, v_s^0)$. The displacement components of a neighboring stable state differ by $(u_0^1, v_0^1, w_b^1, v_s^1)$ with respect to the equilibrium position.

Thus, the total displacements of a neighboring state are

$$u_0 = u_0^0 + u_0^1,\ v_0 = v_0^0 + v_0^1,\ w_b = w_b^0 + w_b^1,\ w_s = w_s^0 + w_s^1 \tag{28}$$

where the superscript 1 refers to the state of stability and the superscript 0 refers to the state of equilibrium conditions.

Substituting Equations (13), (14), (22), (23) and (28) into Equation (19) and integrating by parts and then equating the coefficients of δu_0^1, δv_0^1, δw_b^0 and δw_s^1 to zero, separately, the governing stability equations are obtained for the shear deformation plate theories as

$$\begin{aligned}
&\frac{\partial N_x^1}{\partial x}+\frac{\partial N_{xy}^1}{\partial y}=0\\
&\frac{\partial N_{xy}^1}{\partial x}+\frac{\partial N_y^1}{\partial y}=0\\
&\frac{\partial^2 M_x^{b1}}{\partial x^2}+2\frac{\partial^2 M_{xy}^{b1}}{\partial x\partial y}+\frac{\partial^2 M_y^{b1}}{\partial y^2}+\overline{N}=0\\
&\frac{\partial^2 M_x^{s1}}{\partial x^2}+2\frac{\partial^2 M_{xy}^{s1}}{\partial x\partial y}+\frac{\partial^2 M_y^{s1}}{\partial y^2}+\frac{\partial Q_{xz}^{s1}}{\partial x}+\frac{\partial Q_{yz}^{s1}}{\partial y}+\overline{N}=0
\end{aligned} \tag{29}$$

with

$$\overline{N}=N_x^0\frac{\partial^2(w_b^1+w_s^1)}{\partial x^2}+N_y^0\frac{\partial^2(w_b^1+w_s^1)}{\partial y^2} \tag{30}$$

$$N_x^0=N_y^0=-\frac{\Phi}{1-\nu} \tag{31}$$

and

$$\Phi=\int_{-h/2}^{h/2}E(z)\alpha(z)T\,dz \tag{32}$$

Clearly, when the effect of transverse shear deformation is neglected (ws = 0), the governing equation Eq. (29) yields the governing equation of FGM plate based on the classical plate theory.

2.4. Thermal buckling solution

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Equations (29) for a simply supported FGM sandwich plate. The following boundary conditions are imposed for the present refined plate theory at the side edges:

$$v_0^1=w_b^1=w_s^1=\frac{\partial w_s^1}{\partial y}=N_x^1=M_x^{b1}=M_x^{s1}=0,\ at\ x=0,a \tag{33}$$

$$u_0^1=w_b^1=w_s^1=\frac{\partial w_s^1}{\partial x}=N_y^1=M_y^{b1}=M_y^{s1}=0,\ at\ y=0,b \tag{34}$$

The following approximate solution is seen to satisfy both the differential equation and the boundary conditions

$$\begin{Bmatrix} u_0^1\\ v_0^1\\ w_b^1\\ w_s^1\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}^1\cos\lambda x\sin\mu y\\ V_{mn}^1\sin\lambda x\sin\mu y\\ W_{bmn}^1\sin\lambda x\sin\mu y\\ W_{smn}^1\sin\lambda x\sin\mu y\end{Bmatrix} \tag{35}$$

wehre

U_{mn}^1, V_{mn}^1, W_{bmn}^1, W_{smn}^1 are arbitrary parameters to be determined and $\lambda = m\pi$ a and $\mu = n\pi$ b.

Substituting Equation (35) into Equation (29), one obtains

$$[S]\{\Delta\}=0 \tag{36}$$

where $\{\Delta\}$ denotes the column

and [S] is the symmetric matrix given by

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14}\\ s_{12} & s_{22} & s_{23} & s_{24}\\ s_{13} & s_{23} & s_{33} & s_{34}\\ s_{14} & s_{24} & s_{34} & s_{44}\end{bmatrix} \tag{37}$$

in which:

$$\begin{aligned}
&{}_{11}=A_{11}\lambda^2+A_{66}\mu^2,\quad s_{12}=\lambda\mu(A_{12}+A_{66}),\quad s_{22}=A_{66}\lambda^2+A_{22}\mu^2,\\
&{}_{13}=-\lambda\left[B_{11}\lambda^2+(B_{12}+2B_{66})\mu^2\right]\quad s_{14}=-\lambda\left[B_{11}^s\lambda^2+(B_{12}^s+2B_{66}^s)\mu^2\right]\\
&{}_{23}=-\mu\left[(B_{12}+2B_{66})\lambda^2+B_{22}\mu^2\right]\quad s_{24}=-\mu\left[(B_{12}^s+2B_{66}^s)\lambda^2+B_{22}^s\mu^2\right]\\
&{}_{33}=D_{11}\lambda^4+2(D_{12}+2D_{66})\lambda^2\mu^2+D_{22}\mu^4+N_x^0\left(\lambda^2+\mu^2\right)\\
&{}_{34}=D_{11}^s\lambda^4+2(D_{12}^s+2D_{66}^s)\lambda^2\mu^2+D_{22}^s\mu^4+N_x^0\left(\lambda^2+\mu^2\right)\\
&{}_{44}=H_{11}^s\lambda^4+2(H_{12}^s+2H_{66}^s)\lambda^2\mu^2+H_{22}^s\mu^4+A_{55}^s\lambda^2+A_{44}^s\mu^2+N_x^0\left(\lambda^2+\mu^2\right)\ ,
\end{aligned} \tag{38}$$

By applying the condensation approach to eliminate the in-plane displacements U_{mn}^1 and V_{mn}^1,

Eq. (36) can be rewritten as

$$[S\,]\{\Delta\}=0 \tag{39}$$

where

$$\left[\overline{S}\right]=\begin{bmatrix}\overline{S}_{33} & \overline{S}_{43}\\ \overline{S}_{43} & \overline{S}_{44}\end{bmatrix}\qquad \left[\overline{\Delta}\right]=\begin{Bmatrix}W_{bmn}\\ W_{smn}\end{Bmatrix} \tag{40}$$

and

$$\begin{aligned}
&\bar{s}_{33}=s_{33}-s_{13}\frac{b_1}{b_0}-s_{23}\frac{b_2}{b_0}+N_x^0(\lambda^2+\mu^2),\bar{s}_{34}=s_{34}-s_{14}\frac{b_1}{b_0}-s_{24}\frac{b_2}{b_0}+N_x^0(\lambda^2+\mu^2),\\
&\bar{s}_{43}=s_{34}-s_{13}\frac{b_3}{b_0}-s_{23}\frac{b_4}{b_0}+N_x^0(\lambda^2+\mu^2),\bar{s}_{44}=s_{44}-s_{14}\frac{b_3}{b_0}-s_{24}\frac{b_4}{b_0}+N_x^0(\lambda^2+\mu^2),\\
&b_0=s_{11}s_{22}-s_{12}^2,b_1=s_{13}s_{22}-s_{12}s_{23},b_2=s_{11}s_{23}-s_{12}s_{13},b_3=s_{14}s_{22}-s_{12}s_{24},b_4=s_{11}s_{24}-s_{12}s_{14}
\end{aligned} \tag{41}$$

In the following, the solution of the equation $S_{ij} = 0$ for different types of thermal loading conditions and by using refined plate theory is presented. The temperature change is varied only through-the-thickness.

2.4.1 Buckling of FGM plates under uniform temperature rise

The plate initial temperature is assumed to be T_i . The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is $\Delta T = T_f - T_i$.

Substituting prebuckling forces from Eqs. (32) into the matrix S and setting S = 0 to obtain the nonzero solution, the value of ΔT is found as

$$\Delta T_{cr} = \frac{(1-\nu)}{P(\lambda^2+\mu^2)} \frac{(\bar{s}_{33}\bar{s}_{44} - \bar{s}_{34}\bar{s}_{43})}{(\bar{s}_{33} + \bar{s}_{44} - \bar{s}_{34} - \bar{s}_{43})} \tag{42}$$

where

$$P = \int_{-h/2}^{h/2} E(z)\alpha(z)dz \tag{43}$$

For the case of CPT, the expression of critical buckling temperature can b simplified by setting the shear component of transverse displacement to zero ($w_s = 0$) as

$$\Delta T_{cr} = \frac{(1-\nu)\bar{s}_{33}}{P(\lambda^2+\mu^2)} \tag{44}$$

The critical temperature difference is obtained for the values of m, n that make the preceding expression a minimum. Apparently, when minimization methods are used, critical temperature difference is obtained for m= n=1.

2.4.2. Buckling of FGM plates under non-linear temperature change across the thickness

The functionally graded materials are designed in order to resist high temperature rise by ceramic, so the temperature change will be quite different at the two sides of the FGM structures. When the temperature rises differently at the inner and outer surfaces of the plate, the temperature distribution across the thickness is governed by the steady state heat conduction equation and boundary condition as follows [8]:

$$\frac{d}{dz}\left(K(z)\frac{dT}{dz}\right) = 0 \ , \ T(h/2) = T_c \ , \ T(-h/2) = T_m \tag{45}$$

where K(z) is the coefficient of thermal conduction. Similar to the elasticity and thermal expansion properties, we assume that the thermal conductive coefficient is also a power form function as

$$K(z) = K_{cm}(z/h + 1/2)^k + K_m \tag{46}$$

where

$$K_{cm} = K_c - K_m \tag{47}$$

The solution of Eq. (45) is obtained by means of polynomial series. Taking the first seven terms of the series, the solution for temperature distribution across the plate thickness becomes [8]

$$T(z) = T_m + (T_c - T_m)\eta(z) \tag{48}$$

with

$$\eta(z) = \frac{1}{C}\left[\left(\frac{2z+h}{2h}\right) - \frac{K_{cm}}{(k+1)K_m}\left(\frac{2z+h}{2h}\right)^{k+1} + \frac{K_{cm}^2}{(2k+1)K_m^2}\left(\frac{2z+h}{2h}\right)^{2k+1} - \frac{K_{cm}^3}{(3k+1)K_m^3}\left(\frac{2z+h}{2h}\right)^{3k+1} + \frac{K_{cm}^4}{(4k+1)K_m^4}\left(\frac{2z+h}{2h}\right)^{4k+1} - \frac{K_{cm}^5}{(5k+1)K_m^2}\left(\frac{2z+h}{2h}\right)^{5k+1}\right] \tag{49}$$

$$C = 1 - \frac{K_{cm}}{(k+1)K_m} + \frac{K_{cm}^2}{(2k+1)K_m^2} - \frac{K_{cm}^3}{(3k+1)K_m^3} + \frac{K_{cm}^4}{(4k+1)K_m^4} - \frac{K_{cm}^5}{(5k+1)K_m^5} \tag{50}$$

where $\Delta T = T_c - T_m$ is defined as the temperature difference between ceramic-rich and metal- rich surfaces of the plate. Similar to the previous loading case, the critical buckling temperature change ΔT_{cr} can be deduced, for the refined plate theory, as

$$\Delta T_{cr} = \frac{\dfrac{(1-\nu)}{(\lambda^2+\mu^2)} \dfrac{(\bar{s}_{33}\bar{s}_{44} - \bar{s}_{34}\bar{s}_{43})}{(\bar{s}_{33} + \bar{s}_{44} - \bar{s}_{34} - \bar{s}_{43})} - PT_m}{H} \tag{51}$$

where

$$H = \int_{-h/2}^{h/2} E(z)\alpha(z)\eta(z)dz \tag{52}$$

For the case of CPT, the expression of critical buckling temperature can be simplified by setting the shear component of transverse displacement to zero ($w_s = 0$) as

$$\Delta T_{cr} = \frac{\dfrac{(1-\nu)\bar{s}_{33}}{(\lambda^2+\mu^2)} - PT_m}{H} \tag{53}$$

3. RESULTS AND DISCUSSION

3.1 Validation of the results

The study, here, is focused on the thermal buckling of functionally graded rectangular plates. In this section, we use the above formulation to investigate the effect of parameters like gradient index, the plate aspect ratio and the relative thickness on thermoelastic stability behavior of functionally graded rectangular plates.

Fig. 2 shows the variation of the volume fractions of ceramic through the dimensionless thickness z/h with different values of volume fraction index n. If k= 0 then the plate reduces to a pure ceramic plate. As the volume fraction index k increases, the ceramic volume fraction decreases. Plates are made of a mixture of ceramics and metals; and it is assumed that their composition is gradual and that they are smoothly varied from the ceramic-rich top surface of the plate (z=+h/2) to the metal rich bottom surface (z= - h/2).

The Young's modulus, conductivity and the coefficient of thermal expansion for alumina is Ec = 380 GPa, Kc= 10.4 W/mK, αc = 7.4x10-6 (1/°C), and for aluminium is Em=70 GPa, Km=204 W/mK, αm= 23x10-6 (1/°C), respectively. Poisson's ratio is chosen as ν= 0.3.

The temperature variation in the thickness direction as per Eq. (48) with the volume fraction index is presented in Fig. 3 for Tc/Tm = 15 and it can be noted that the temperature variation through the thickness of functionally graded plate is non-linear compared to those of pure ceramic case (k = 0). Based on new four variable refined plate theory, to be adequate to model of plate for thermal stability study as observed from Table 1.

Also the results are found to be in very good agreement with the available results [11, 27] as seen from this table. The plate is of uniform thickness.

Table 1 Comparison of the critical buckling temperature of a square FGM under uniform temperature.

k	Method	a/h					
		10	20	40	60	80	100
0	Javaheri et al [27]	1617.484	421.516	106.492	47.424	26.693	17.088
	Chen et al [11]	1593.902	419.739	106.370	47.396	26.684	17.084
	Present results	1618.682	421.535	106.494	47.423	26.694	17.089
1	Javaheri et al [27]	757.891	196.257	49.500	22.037	12.402	7.939
	Chen et al [11]	746.076	195.391	49.4422	22.024	12.398	7.937
	Present results	758.396	196.265	49.5017	22.037	12.403	7.940
5	Javaheri et al [27]	678.926	178.528	45.213	20.144	11.340	7.260
	Chen et al [25]	672.509	177.994	45.1702	20.132	11.335	7.258
	Present results	679.310	178.535	45.214	20.144	11.340	7.261
10	Javaheri et al [27]	692.519	183.141	46.455	20.703	11.657	7.462
	Chen et al [11]	687.042	182.649	46.4077	20.688	11.649	7.459
	Present results	692.695	183.144	46.455	20.703	11.656	7.463

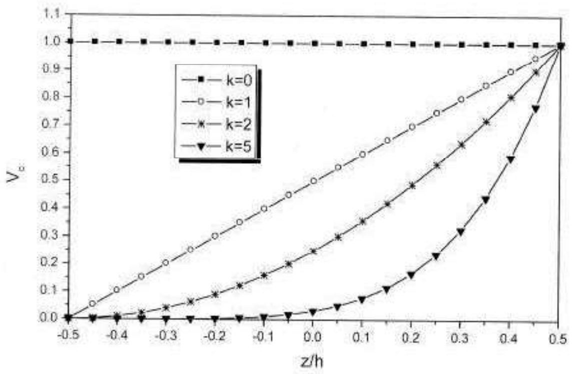

Fig. 4 investigates the critical buckling temperature of homogeneous and FGM plates versus the side-to thickness ratio a / h under uniform temperature load for different values of volume fraction exponent k (a/b=1). It is seen that the critical temperature difference decreases monotonically as the relative thickness a/h increases. In addition, the critical temperature change increases as volume fraction index k is increased. This is because for FGMs, as the volume fraction index is increased, the contained quantity of ceramic increases.

Fig. 5 shows the variation trend of critical temperature difference with respect to the relative thickness a/h for different values of material gradient index k(a/b=1) under non-linear temperature load. The responses are very similar, however, the critical temperature gradient under non linear temperature rise is higher than that under uniforı temperature rise.

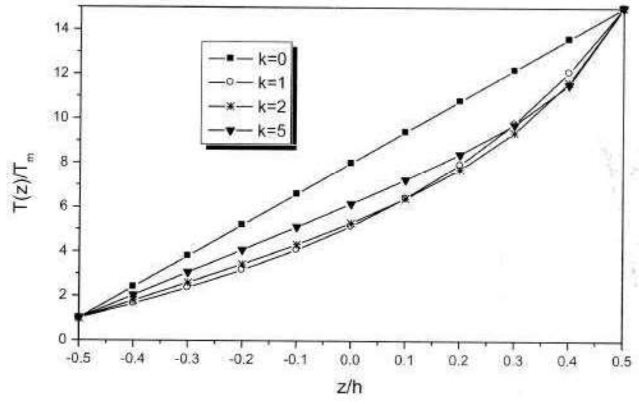

Fig. 3. Variation of temperature through thickness

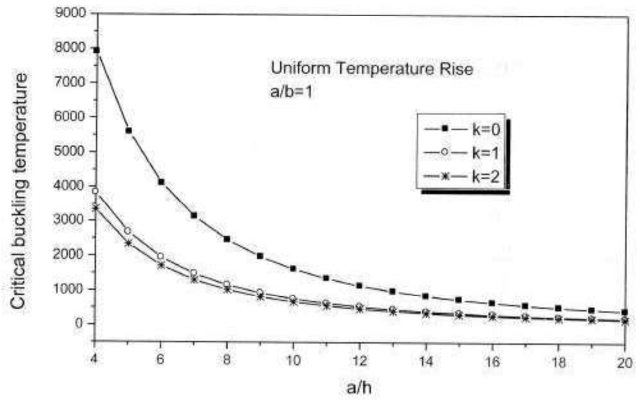

Fig.4. Critical buckling temperature due to uniform temperature rise versus the side-to thickness ratio a / h.

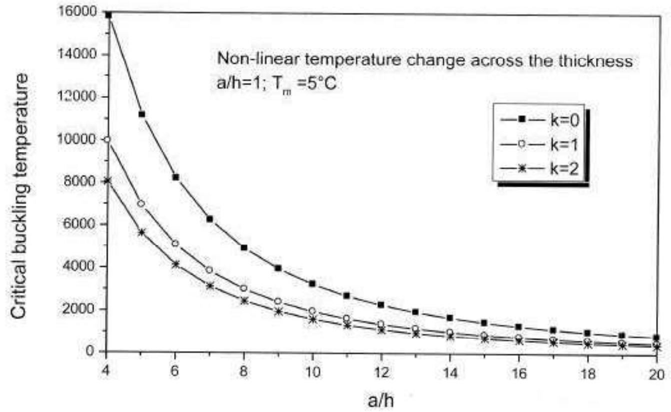

Fig.5. Critical buckling temperature due to non-linear temperature change across the thickness versus the side-tc thickness ratio a / h.

4. CONCLUSIONS

The asymmetric thermal buckling of functionally graded rectangular plates has been analyzed through new four variable refined plate theory. The material properties are assumed tc be varied through the thickness direction based on power law distribution. The effect of parameters like gradient index, the relative thickness on thermoelastic stability behavior has een investigated for functionally graded rectangular plates. rom the detailed parametric study, the following observa-ons can be made:

With the increase in volume fraction index, the critical buckling temperature decrease.

For FGM plates under nonlinear temperature change across the thickness, the critical buckling temperature is greater than that of uniform temperature distribution.

For uniform temperature rise when the power law index k is greater than 2, the critical buckling temperature changes very slowly by changing k.

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